hw0

Coding the Matrix, Summer 2013

Please fill out the stencil file named "`hw0.py`". While we encourage you to complete the Ungraded Problems, they do not require any entry into your stencil file.

Python Comprehension Problems

Write each of the following 3 procedures using a comprehension:

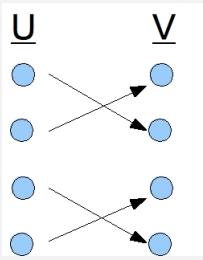
Problem 1: `myFilter(L, num)`
input: list of numbers and a number.
output: list of numbers not containing a multiple of `num`.
example: given `list = [1,2,4,5,7]` and `num = 2`, return `[1,5,7]`.

Problem 2: `myLists(L)`
input: list `L` of non-negative integers.
output: a list of lists: for every element x in `L` create a list containing $1, 2, \ldots, x$.
example: given `[1,2,4]` return `[[1],[1,2],[1,2,3,4]]`.

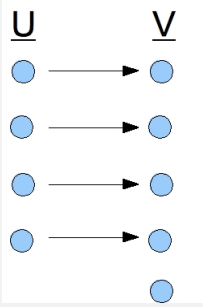
Problem 3: `myFunctionComposition(f,g)`
input: two functions f and g, represented as dictionaries, such that $g \circ f$ exists.
output: dictionary that represents a function $g \circ f$.
example: given $f = \{0\text{:'a'}, 1\text{:'b'}\}$ and $g = \{\text{'a':'apple', 'b':'banana'}\}$, return $\{0\text{:'apple'}, 1\text{:'banana'}\}$.

Functional Inverses

Ungraded Problem: Is the following function invertible? If yes, explain why. If not, can you change domain and/or codomain of the function to make it invertible?



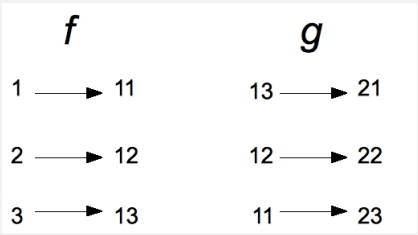
Ungraded Problem: Is the following function invertible? If yes, explain why. If not, can you change domain and/or codomain of the function to make it invertible?



Functional Composition

Ungraded Problem: Let $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = abs(x)$. Is there a choice of domain and co-domain for the function $g(x)$ with rule $g(x) = \sqrt{x}$ such that $g \circ f$ is defined? If so, specify it. If not, explain why not. Could you change domain and/or codomain of f or g so that $g \circ f$ will be defined?

Ungraded Problem: Consider functions f and g in the following figure:



Is $f \circ g$ defined? If so, draw it, otherwise explain why not.

Complex Addition Practice

Problem 4: Each of the following asks for the sum of two complex numbers. For each, find the solution.

a. $(3 + 1\mathbf{i}) + (2 + 2\mathbf{i})$

b. $(-1 + 2\mathbf{i}) + (1 - 1\mathbf{i})$

c. $(2 + 0\mathbf{i}) + (-3 + .001\mathbf{i})$

d. $4(0 + 2\mathbf{i}) + (.001 + 1\mathbf{i})$

GF(2) Arithmetic

Problem 5: For each of the following, calculate the answer over $GF(2)$.

a. $1+1+1+0$

b. $1 \cdot 1 + 0 \cdot 1 + 0 \cdot 0 + 1 \cdot 1$

c. $(1+1+1) \cdot (1+1+1+1)$

Python loop problems

For procedures in the following problems, use the following format:

```
def <ProcedureName>(L):
    current = ...
    for x in L:
        current = ...
    return current
```

The value your procedure initially assigns to `current` turns out to be the return value in the case when the input list `L` is empty. This provides us insight into how the answer should be defined in that case. Note: You are **not** allowed to use Python built-in procedures `sum(·)` and `min(·)`.

Summing numbers in a list

Problem 6: `mySum(L)`
Input: list of numbers
Output: sum of numbers in the list

Multiplying numbers in a list

Problem 7: `myProduct(L)`
Input: list of numbers
Output: product of numbers in the list

Minimum of a list

Problem 8: `myMin(L)`
Input: list of numbers
Output: minimum number in the list

Concatenation of a List

Problem 9: `myConcat(L)`
Input: list of strings
Output: concatenation of all the strings in `L`

Union of Sets in a List

Problem 10: `myUnion(L)`
Input: list of sets
Output: the union of all sets in `L`.

In each of the above problems, the value of `current` is combined with an element of `myList` using some operation $\diamond$. In order that the procedure return the correct result, `current` should be initialized with the *identity element* for the operation $\diamond$, i.e. the value i such that $i \diamond x = x$ for any value x.

It is a consequence of the structure of the procedure that, when the input list is empty, the output value is the initial value of `current` (since in this case the body of the loop is never executed). It is convenient to define this to be the correct output!

Identity Values

Ungraded Problem: Keeping in mind the comments above, what should be the answer for each of the following?

1. the sum of the numbers in an empty set;
2. the product of the numbers in an empty set;
3. the minimum of the numbers in an empty set;
4. the concatenation of an empty list of strings;
5. the union of an empty list of sets

What goes wrong when we try to apply this reasoning to define the intersection of an empty list of sets?